Joint Encoding and Node-Pair Grouping for Physical-Layer Network Coding

Fei Huang*, Shiqiang Wang†, Qingyang Song*§, Lei Guo* and Abbas Jamalipour‡
*School of Information Science and Engineering, Northeastern University, Shenyang 110819, P. R. China
†Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ, United Kingdom
‡School of Electrical and Information Engineering, University of Sydney, NSW, 2006, Australia
Email: huangfeinf@gmail.com, shiqiang.wang11@imperial.ac.uk, songqingyang@ise.neu.edu.cn,
guolei@ise.neu.edu.cn, a.jamalipour@ieee.org

***Abstract*—Physical-layer network coding (PNC) is a promising technique to improve the performance of wireless networks. In a traditional setting, two packets are encoded with PNC. However, encoding three or more packets with PNC may further enhance the performance. A straightforward method to achieve multiple packet encoding is to further process the superposed signal from different source node-pairs at the relay, and broadcast a jointly encoded packet to the destinations. After receiving the jointly encoded packet, each destination can decode the packet it intends to receive, based on the packet transmitted by itself and overheard packets from other node-pairs. An important issue that arises is how to determine which node-pairs should be encoded together and how to jointly encode these packets. This paper aims to solve this issue. We first propose a joint encoding method for multiple node-pairs, and then propose a method to determine the grouping of different node-pairs so that node-pairs in the same group can perform joint encoding. The proposed scheme is not restricted to a particular signal constellation, and, in principle, it can be applied to any type of signal constellations. Simulation results show that the proposed scheme can bring throughput improvement compared to schemes which only encode two packets together, the resulting throughput is also close to the optimum.**

***Index Terms*—Constellation mapping, denoise-and-forward (DNF), physical-layer network coding (PNC), quadrature amplitude modulation (QAM), wireless networks.**

I. Introduction

Wireless physical-layer network coding (PNC) makes use of the superposition nature of electromagnetic fields to improve the throughput of wireless networks [1], [2]. The source nodes transmit their packets simultaneously to the relay, the relay processes the superposed signal and sends an encoded version of the superposed signal to the destinations. Then, from the encoded packet, each destination decodes the packet it intends to receive, based on its knowledge of the other simultaneously transmitted packet(s) that is obtained either from overhearing or because itself is the source of the packet. The process of PNC can be divided into two communication phases, namely, the multiple access (MA) phase where the involved source nodes transmit their packets to the relay at the same time, and the broadcast (BC) phase where the relay sends the encoded packet to the destinations.

In a traditional setting of PNC, two source nodes, two destination nodes, and one relay are involved in the PNC process.

§ The corresponding author of this paper is Qingyang Song.

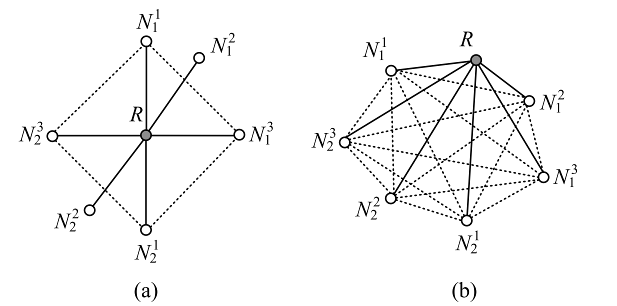

Fig. 1. Example network topology with three node-pairs, the solid lines represent links connected to the relay, the dotted lines represent overhearing links: (a) when two node-pairs can be grouped together, (b) when three node-pairs can be grouped together.

The destination nodes may or may not be source nodes at the same time. As discussed in [3], the involvement of three or more source nodes that transmit at the same time generally requires full-duplex transceivers or interference cancellation techniques. A recent study [4] has proposed a mechanism to pre-rotate the signal constellations at the source nodes before transmitting, so that the superposed signals at the relay can still be effectively separated when three source nodes transmit at the same time, which is similar with interference cancellation. On the other hand, when we only consider the involvement of three or more destinations, a more straightforward approach applies. As in the approach proposed in [5], only two source nodes transmit simultaneously to the relay in each timeslot, these two source nodes that transmit at the same time form up a source *node-pair*, and all source node-pairs transmit to the relay sequentially. Then, the relay performs a joint encoding on all received superposed signals and broadcasts the encoded packet to all destinations, and the destinations extract their intended packets. By this means, the number of necessary BC timeslots is reduced.

In the example as shown in Fig. 1 and also in the remaining discussions, nodes N_1^i and N_2^i form up a node-pair, and we denote the node-pair by $N_1^i \leftrightarrow N_2^i$, where $i \in \{1, 2, ..., K\}$ and K is the total number of node-pairs under consideration. For each i, nodes N_1^i and N_2^i exchange packets with each other, i.e. there is a bidirectional flow among N_1^i and N_2^i. All the packets are relayed by the relay R. To accomplish data transmission for all nodes, every scheduling round consists of

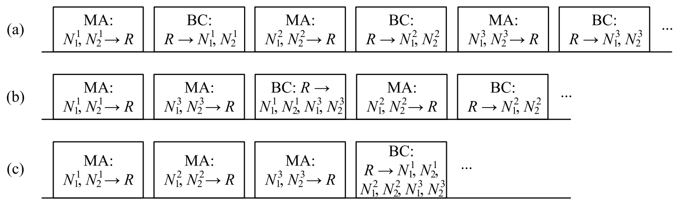

Fig. 2. Example timing diagram of packet transmission in one scheduling round with three node-pairs: (a) when no node-pair can be grouped together, (b) when two node-pairs can be grouped together, (c) when three node-pairs can be grouped together.

K MA timeslots to serve each of the K node-pairs once, and a specific number of BC timeslots, which depends on the joint encoding opportunity and is at least one and at most K.

In Fig. 1(a). we have a total of $K = 3$ node-pairs. When using conventional PNC, we need a total of $2K = 6$ timeslots to accomplish data exchange, as shown in Fig. 2(a). If there exist overhearing links between the nodes in a certain node-pair (e.g. $N_1^1 \leftrightarrow N_2^1$) and nodes that belong to other node-pairs (e.g. $N_1^3 \leftrightarrow N_2^3$), when the nodes N_1^1 and N_2^1 transmit simultaneously in the MA phase, both the relay R and the nodes in the node-pair $N_1^3 \leftrightarrow N_2^3$ can receive/overhear a superposed signal. If we schedule the node-pair $N_1^3 \leftrightarrow N_2^3$ to transmit in the next timeslot instead of letting R broadcast the encoded packet, both the relay R and the nodes in the node-pair $N_1^1 \leftrightarrow N_2^1$ can receive/overhear a superposed signal from the node-pair $N_1^3 \leftrightarrow N_2^3$. In this case, the relay R may *jointly encode* the two superposed signals with a specific method so that the four involved nodes can extract the intended packets after receiving the encoded packet. As a result, the number of BC timeslots is reduced, as shown in Fig. 2(b). For the above case, in this paper, we say that node-pairs $N_1^1 \leftrightarrow N_2^1$ and $N_1^3 \leftrightarrow N_2^3$ can be *grouped* together. When all the three node-pairs can be grouped together as in Fig. 1(b), we only need $2K - 2 = 4$ timeslots (Fig. 2(c)).

Note that in Fig. 1(b), the relay R is not really necessary when judging from the network topology, because all nodes can communicate with each other. However, without the relay, we still need $2K = 6$ timeslots to transmit all data packets to their destinations when using half-duplex communication systems. After adding a relay, we can reduce the number of necessary timeslots and therefore improve average throughput. It follows that, when combining with node-pair grouping and joint encoding, PNC can still be beneficial when nodes are reachable within single hop. When K is large and the signal qualities among nodes are good enough, the number of necessary timeslots to accomplish one round of packet exchange approaches K, which is the same with full-duplex communication systems. The inclusion of a relay even when source and destination nodes are within their communication range can also be found in many practical systems, such as wireless local area networks (WLAN) with access points (AP) or cellular networks with base-stations. The relay also acts as a coordinator in most of these networks.

Given the importance of node-pair grouping and joint encoding, it is desirable to develop a mechanism to adaptively decide which node-pairs should be grouped together and what encoding function should be selected. To the best of our knowledge, [5] is the only work which considered this issue in the literature, where the joint encoding of binary phase-shift keying (BPSK) signals were studied. However, [5] did not consider how to adaptively determine the node-pair groups and also did not discuss how to encode signals with higher level modulations, e.g. quadrature phase-shift keying (QPSK) or general multi-level quadrature amplitude modulation (MQAM). For high level modulations, the conventional XOR-mapping is not applicable due to ambiguity [6]. Furthermore, it is impractical to synchronize the signals both at the relay and at the overhearing nodes. Hence, a more sophisticated constellation mapping scheme that supports asynchronous signals, such as [7], is necessary and it needs to be extended to support joint encoding.

To tackle these issues, in this paper, we first discuss the requirements of the encoding functions to support joint encoding. Based on this discussion, we propose a joint encoding and node-pair grouping scheme, which adaptively selects the encoding functions and node-pair groups according to the channel coefficients. The goal of this paper is to determine the grouping of node-pairs and the encoding method within each group, to minimize the number of timeslots that is needed to accomplish one round of data exchange for all K node-pairs, subject to a given bit-error rate (BER) constraint.

The remainder of the paper is organized as follows. Section II describes the system model. Section III discusses the encoding function design. The node-pair grouping method is discussed in Section IV. Section V shows simulation results and Section VI draws conclusions.

II. SYSTEM MODEL

We consider a network topology with one relay and an arbitrary number of node-pairs that are connected to the relay, as in Fig. 1. We assume that a central scheduler is present which is aware of all channel gains between nodes and schedules the transmission and encoding of different nodes. In practice, the functions of this central scheduler can be performed by the relay R, such as in [8]. The design of an explicit scheduling mechanism is beyond the scope of this paper, but we consider a scheduling mechanism where each node-pair is served once in each scheduling round, as in Fig. 2. BC timeslots in which the relay R broadcasts to a set of node-pairs always comes after all MA timeslots for the involved node-pairs. There may be different number of BC timeslots in each scheduling round due to different joint encoding opportunities. We consider flat fading channels in this paper.

Define an index set $\mathcal{G} \subseteq \{1, 2, ..., K\}$ which represents a group of node-pairs. For any $g \in \mathcal{G}$, we say that node-pair $N_1^g \leftrightarrow N_2^g$ belongs to group $\mathcal{G}$, and the set $\mathcal{G}$ contains the indexes of all node-pairs in the group. The number of node-pairs in group $\mathcal{G}$ is denoted by $|\mathcal{G}|$. Let $S_{N_j^i} \in \mathcal{M}$ ($j = 1, 2$) denote the symbol from node N_j^i, where $\mathcal{M}$ is the set of possible symbols, and let $X_{N_j^i}$ denote the corresponding signal. When a particular node-pair $N_1^{g_t} \leftrightarrow N_2^{g_t}$ ($g_t \in \mathcal{G}$) that belongs to group $\mathcal{G}$ transmit signals, the relay R and each node

$N_j^{g_r}$ $(g_r \neq g_t, \forall g_r \in \mathcal{G})$ in each of the remaining node-pairs that belongs to $\mathcal{G}$ receive a superposed signal[1]. The received signals are respectively:

$$Y_R^{\mathrm{MA}} = H_{N_1^{g_t},R} X_{N_1^{g_t}} + H_{N_2^{g_t},R} X_{N_2^{g_t}} + Z_n \,, \quad (1)$$

$$Y_{N_j^{g_r}}^{\mathrm{MA}} = H_{N_1^{g_t},N_j^{g_r}} X_{N_1^{g_t}} + H_{N_2^{g_t},N_j^{g_r}} X_{N_2^{g_t}} + Z_n \,, \quad (2)$$

where $H_{N_j^{g_t},R}$ (or $H_{N_j^{g_t},N_j^{g_r}}$) is the channel coefficient from node $N_j^{g_t}$ to R (or $N_j^{g_r}$), and Z_n is the noise.

After receiving the superposed signal, each of the relay R and node $N_j^{g_r}$ estimate the originally transmitted symbols $(S_{N_1^{g_t}}, S_{N_2^{g_t}})$ as $(\hat{S}_{N_1^{g_t}}, \hat{S}_{N_2^{g_t}})_R$ or $(\hat{S}_{N_1^{g_t}}, \hat{S}_{N_2^{g_t}})_{N_j^{g_r}}$. We consider the case where the original symbols are equiprobable and the noise is Gaussian, hence we use the minimum distance criterion [9] for estimation. It is not always possible to correctly decode the individual symbols $S_{N_1^{g_t}}$ and $S_{N_2^{g_t}}$ from the superposed signal, which is also not necessary as long as the encoding function is properly designed so that each node can finally obtain the symbol that it intends to receive.

After all the MA timeslots for group $\mathcal{G}$, the relay R has received a set of symbols $\hat{\mathcal{S}}_R = \left\{(\hat{S}_{N_1^g}, \hat{S}_{N_2^g})_R : \forall g \in \mathcal{G}\right\}$; and similarly, each node N_j^g has overheard a set of symbols $\hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g} = \left\{(\hat{S}_{N_1^{g_t}}, \hat{S}_{N_2^{g_t}})_{N_j^g} : g_t \neq g, \forall g_t \in \mathcal{G}\right\}$, i.e. symbols from nodes excluding its own node-pair. The relay R encodes the received set of symbols $\hat{\mathcal{S}}_R$ into a new symbol $C\left(\hat{\mathcal{S}}_R\right)$, where the encoding mapping is $C : (\mathcal{M} \times \mathcal{M})^{|\mathcal{G}|} \to \mathcal{M}_C$ and $\mathcal{M}_C$ is the set of all possible encoded symbols. Then, it broadcasts the encoded symbol to all nodes in $\mathcal{G}$, and each node N_j^g receives the signal:

$$Y_{N_j^g}^{\mathrm{BC}} = H_{R,N_j^g} X_C + Z_n \,, \quad (3)$$

where X_C denotes the signal that carries the encoded symbol $C\left(\hat{\mathcal{S}}_R\right)$, and H_{R,N_j^g} is the channel coefficient from node R to N_j^g. From the received signal, each node N_j^g can estimate the value of $C\left(\hat{\mathcal{S}}_R\right)$, and decode the intended symbol based on $C\left(\hat{\mathcal{S}}_R\right)$ and the overheard symbols in $\hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}$. The design of the encoding function $C(\cdot)$ will be discussed in Section III.

To consider the grouping of all K node-pairs, we define a set $\mathcal{P} = \{\mathcal{G}_1, \ldots, \mathcal{G}_{|\mathcal{P}|}\}$, where $1 \leq |\mathcal{P}| \leq K$, which represents a partition with $|\mathcal{P}|$ groups. All the groups are subsets of $\{1, 2, \ldots, K\}$ and should satisfy the following two properties: $\mathcal{G}_1 \cup \cdots \cup \mathcal{G}_{|\mathcal{P}|} = \{1, 2, \ldots, K\}$ and $\mathcal{G}_1 \cap \cdots \cap \mathcal{G}_{|\mathcal{P}|} = \phi$. The node-pair grouping problem is formulated as finding the most appropriate partition $\mathcal{P}$, which will be further discussed in Section IV.

III. Encoding Node-Pairs in the Same Group

In this section, we focus on designing an appropriate joint encoding method for node-pairs in the same group.

[1]Nodes that belong to other groups may also receive the signal, but they neglect the signal because it is not related to them.

A. Goals for the Encoding Function

The encoding function $C(\cdot)$ must ensure that each node must be able to obtain the symbol from the other node in its own node-pair, under the following conditions: it knows the symbol transmitted by itself, has overheard the transmission of all other node-pairs in group $\mathcal{G}$, and has also received the encoded symbol from the relay R. Formally, the design of $C(\cdot)$ aims to achieve the following: for each N_j^g, given $S_{N_j^g}$ (its own signal), $C\left(\hat{\mathcal{S}}_R\right)$, and $\hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}$, it must be able to decode $S_{N_{\bar{j}}^g}$, where $\bar{j}$ denotes the opposite of j, i.e. if $j = 1$ then $\bar{j} = 2$, if $j = 2$ then $\bar{j} = 1$.

When there is only one node-pair in group $\mathcal{G}$, in which case $\hat{\mathcal{S}}_R$ contains only one pair of symbols, the goal can be achieved if the encoding function $C(\cdot)$ satisfies the exclusive law as discussed in [7]. For the general case with multiple node-pairs in $\mathcal{G}$, the exclusive law can be generalized as Lemma 1.

Lemma 1: To ensure that the intended symbol can be successfully decoded at the destination, the encoding function $C(\cdot)$ must satisfy: for any possible set of symbol-pairs $\mathcal{S}_0 = \left\{(s_{N_1^g}, s_{N_2^g}) \in \mathcal{M} \times \mathcal{M} : \forall g \in \mathcal{G}\right\}$ and $\mathcal{S}'_0 = \left\{(s'_{N_1^g}, s'_{N_2^g}) \in \mathcal{M} \times \mathcal{M} : \forall g \in \mathcal{G}\right\}$ which have the following properties: 1) there exists $j_0 \in \{1, 2\}$ and $g_0 \in \mathcal{G}$, such that $s_{N_{j_0}^{g_0}} \neq s'_{N_{j_0}^{g_0}}$; 2) for all $j \neq j_0$ or $g \neq g_0$, we have $s_{N_j^g} = s'_{N_j^g}$; then we must have $C(\mathcal{S}_0) \neq C(\mathcal{S}'_0)$.

Proof: Suppose the condition is not satisfied, then for a particular node N_j^g, given $S_{N_j^g}$, $C\left(\hat{\mathcal{S}}_R\right)$, and $\hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}$, it cannot determine the value of $S_{N_{\bar{j}}^g}$ because different values of $S_{N_{\bar{j}}^g}$ may be mapped to the same value of $C\left(\hat{\mathcal{S}}_R\right)$. On the other hand, if the condition is satisfied, N_j^g can determine the value of $S_{N_{\bar{j}}^g}$ because there exists a one-to-one mapping between $S_{N_{\bar{j}}^g}$ and $C\left(\hat{\mathcal{S}}_R\right)$, when $S_{N_j^g}$ and $\hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}$ are given. ■

The design of $C(\cdot)$ should also attempt to minimize the BER. A bit error exists at the relay R when the actual transmitted symbols $\mathcal{S} \neq \hat{\mathcal{S}}_R$ and at the same time $C(\mathcal{S}) \neq C\left(\hat{\mathcal{S}}_R\right)$, and a bit error exists at node N_j^g when $\mathcal{S} \neq \mathcal{S}(g) \cup \hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}$ and at the same time $C(\mathcal{S}) \neq C\left(\mathcal{S}(g) \cup \hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}\right)$, where $\mathcal{S}(g)$ denotes the symbol-pair corresponding to node-pair g in $\mathcal{S}$ (note that $\hat{\mathcal{S}_{\mathcal{O}}}_{N_j^g}$ excludes the symbol-pair from node-pair g). Therefore, the encoding function $C(\cdot)$ should be designed to avoid the occurrence of such errors.

Further, the design of $C(\cdot)$ should attempt to minimize the cardinality $|\mathcal{M}_C|$ of the set of encoded symbols $\mathcal{M}_C$.

B. Encoding Function Design

In Section III-A, we discussed the goals of the joint encoding function $C(\cdot)$ which processes over all the involved symbols. Considering that signals from two nodes in the same node-pair are simultaneously transmitted and a superposed signal is received at the relay R and end nodes N_j^g, we need to focus on the signal level characteristics to consider the encoding of symbols from nodes in the same node-pair. However, signals from different node-pairs are separately transmitted, so we can use a higher level mechanism to encode the symbols from different node-pairs. It follows that we can decompose the encoding function $C(\cdot)$ into $|\mathcal{G}|$ sub-functions $C_g(\cdot)$, each of which only processes the symbol-pair $(S_{N_1^g}, S_{N_2^g})$. Then, a higher level sub-function $C_0\left(C_{g_1}(\cdot), C_{g_2}(\cdot), ..., C_{g_{|\mathcal{G}|}}(\cdot)\right)$, where $g_1, g_2, ..., g_{|\mathcal{G}|}$ represent all elements in $\mathcal{G}$, processes over each $C_g(\cdot)$. For this construction, it can be easily derived from Lemma 1 that the symbol can be successfully decoded at the destination when the exclusive law satisfies for each $C_g(\cdot)$ (with a pair of symbols as arguments) and the generalized exclusive law satisfies for $C_0(\cdot)$ (with all $C_g(\cdot)$ as arguments). Also, the goal of minimizing BER is approached if we attempt to minimize the error probability in the design of each $C_g(\cdot)$, because the high level sub-function $C_0(\cdot)$ does not deal with signal level issues, hence it does not affect the BER. To minimize the BER in the design of $C_g(\cdot)$, we need to consider the shape of the superposed constellation which may vary with the channel coefficients. Therefore, $C_g(\cdot)$ needs to be adaptively determined based on the instant channel condition, which will be further discussed in Section III-D.

C. High Level Sub-Function

A simple and effective way to design $C_0(\cdot)$ to satisfy the generalized exclusive law is using modulo operation, i.e.

$$C_0\left(C_{g_1}(\cdot), ..., C_{g_{|\mathcal{G}|}}(\cdot)\right) = \left(\sum_{g\in\mathcal{G}} C_g(\cdot)\right) \bmod \left(\max\left\{\left|\mathcal{M}_{C_g}\right| : \forall g \in \mathcal{G}\right\}\right), \quad (4)$$

where $\left|\mathcal{M}_{C_g}\right|$ denotes the cardinality of the set of encoded symbols when encoding with $C_g(\cdot)$. The $\bmod \left(\max\left\{\left|\mathcal{M}_{C_g}\right| : \forall g \in \mathcal{G}\right\}\right)$ operation ensures the generalized exclusive law and keeps the cardinality of the encoded symbol set $\mathcal{M}_C$ as small as possible, and we have $|\mathcal{M}_C| = \max\left\{\left|\mathcal{M}_{C_g}\right| : \forall g \in \mathcal{G}\right\}$.

With the definition of $C_0(\cdot)$ as in (4), when an end node $N_j^{g_t}$ wants to decode its intended symbol $S_{N_{\bar{j}}^{g_t}}$, it can first obtain

$$C_{g_t}(\cdot) = \left(C_0(\cdot) - \sum_{g_r\neq g_t, \forall g_r\in\mathcal{G}} C_{g_r}(\cdot)\right) \bmod \left(\max\left\{\left|\mathcal{M}_{C_g}\right| : \forall g\in\mathcal{G}\right\}\right), \quad (5)$$

where it calculates the values of $C_{g_r}(\cdot)$ from corresponding overheard symbols in $\hat{\mathcal{S}}_{\mathcal{O}N_j^{g_t}}$, and $C_0(\cdot)$ is sent by the relay R to the end nodes in the BC phase. Note that we assume that $C_0(\cdot)$ and $C_g(\cdot), \forall g \in \mathcal{G}$ are known by the relay R and all nodes in group $\mathcal{G}$. Then, it can obtain the intended symbol from $C_{g_t}(\cdot)$.

D. Low Level Sub-Function

In this subsection, we focus on the design of $C_g(\cdot)$. For a node-pair $N_1^{g_t} \leftrightarrow N_2^{g_t}, g_t \in \mathcal{G}$, we consider the sub-function $C_{g_t}(\cdot)$ which deals with the symbol-pair $(S_{N_1^{g_t}}, S_{N_2^{g_t}})$. As the minimum distance criterion [9] is adopted for estimation, an erroneously estimated symbol-pair $(\hat{S}_{N_1^{g_t}}, \hat{S}_{N_2^{g_t}}) \neq (S_{N_1^{g_t}}, S_{N_2^{g_t}})$ is generally close to the actual symbol-pair $(S_{N_1^{g_t}}, S_{N_2^{g_t}})$ in the constellation of the superposed signal. If all symbols-pairs near to $(S_{N_1^{g_t}}, S_{N_2^{g_t}})$ are mapped to the same encoded symbol, and satisfy the exclusive law at the same time, the end nodes can still correctly decode the intended symbols when $(\hat{S}_{N_1^{g_t}}, \hat{S}_{N_2^{g_t}}) \neq (S_{N_1^{g_t}}, S_{N_2^{g_t}})$ and $C_{g_t}(\hat{S}_{N_1^{g_t}}, \hat{S}_{N_2^{g_t}}) = C_{g_t}(S_{N_1^{g_t}}, S_{N_2^{g_t}})$. Therefore, similarly with [7], we attempt to map symbol-pairs which correspond to points that are close to each other in the superposed constellation into identical (encoded) symbols, as long as the exclusive law is satisfied among these symbol-pairs. We denote the set of symbol-pairs that are mapped into an identical symbol as a *cluster*, and the distances among superposed constellation points in different clusters are related to the BER. Different from [7], we consider distances among superposed constellation points at both the relay and each overhearing node, because wrong estimation at the overhearing node can also lead to bit error for the particular node. The superposed constellations at the relay and overhearing nodes can be known when the channel coefficients between nodes and the initial phases of the signals are given.

The algorithm for determining $C_{g_t}(\cdot)$ is shown in Algorithm 1, in which $N_1^{g_t} \leftrightarrow N_2^{g_t}$ denotes the transmitting node-pair of group $\mathcal{G}$. The channel coefficient matrix $\mathbf{H}$, modulation type (determines the possible number $|\mathcal{M}|$ of symbols that $N_j^{g_t}$ may send out), and overhearing nodes $N_1^{g_r} \leftrightarrow N_2^{g_r}, g_r \neq g_t, \forall g_r \in \mathcal{G}$ are given. We can know that the number of superposed constellations under consideration is $2|\mathcal{G}| - 1$ (with $|\mathcal{G}| - 1$ node-pairs plus one relay) and the number of constellation points in each superposed constellation is $|\mathcal{M}|^2$. We use $(s_{N_1^{g_t}}, s_{N_2^{g_t}})$ to denote a particular pair of symbols. The algorithm initially forms a cluster for each symbol-pair independently. After that, it calculates the distances between any two superposed constellation points at both the relay R and each overhearing node $N_j^{g_r}, g_r \neq g_t, \forall g_r \in \mathcal{G}$ (which is denoted by $d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)$, where $n \in \{R\} \cup \{N_1^{g_r}, N_2^{g_r} : g_r \neq g_t, \forall g_r \in \mathcal{G}\}$ is the node under consideration), and stores all the distances into a distance set D. The "while" loop in Algorithm 1 is used to allocate symbols into different clusters. Suppose that $d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)$ is the minimum value in D, we check the exclusive law for symbol-pairs $(s_{N_1^{g_t}}, s_{N_2^{g_t}})$ and $(s'_{N_1^{g_t}}, s'_{N_2^{g_t}})$. If satisfied, the clusters that contain $(s_{N_1^{g_t}}, s_{N_2^{g_t}})$ and $(s'_{N_1^{g_t}}, s'_{N_2^{g_t}})$ are merged together. If not, $d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)$ is a distance between different clusters. We save the distance into d_{min} at the first time when the exclusive law is not satisfied. Then, we compare d_{min} with a threshold d_{th} (which is decided by the BER constraint). If $d_{min} < d_{th}$, the algorithm returns FALSE and stops running. It means that node-pairs in this group cannot be jointly encoded, and we say that group $\mathcal{G}$ is *infeasible*. If $d_{min} \geq d_{th}$, group $\mathcal{G}$ is *feasible*, and the algorithm goes through the whole process and finally returns TRUE. The return value is used in the node-pair grouping algorithm which will be described in Section IV. For a particular combination of two symbol-pairs $(s_{N_1^{g_t}}, s_{N_2^{g_t}})$

Algorithm 1 Generating Low Level Sub-function $C_{g_t}(\cdot)$

1: Given overhearing nodes $N_1^{g_r} \leftrightarrow N_2^{g_r}, g_r \neq g_t, \forall g_r \in \mathcal{G}$
2: Given channel coefficients matrix $\mathbf{H}$ and symbol set $\mathcal{M}$
3: Define a variable for shortest distance between clusters: d_{min}, and initialize with $d_{min} \leftarrow \infty$
4: Define empty set $\mathcal{M}_{C_{g_t}}$ and mapping $C_{g_t} : \mathcal{M} \times \mathcal{M} \rightarrow \mathcal{M}_{C_{g_t}}$
5: **for** all $(s_{N_1^{g_t}}, s_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}$ **do**
6: Generate a cluster set: $U_{(s_{N_1^{g_t}}, s_{N_2^{g_t}})} \leftarrow \{(s_{N_1^{g_t}}, s_{N_2^{g_t}})\}$
7: **for** all $(s'_{N_1^{g_t}}, s'_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}$ **do**
8: **for** all $n \in \{R\} \cup \{N_1^{g_r}, N_2^{g_r} : g_r \neq g_t, \forall g_r \in \mathcal{G}\}$ **do**
9: Compute the distance $d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)$
10: Put it in the distance set: $D \leftarrow D \cup \{d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)\}$
11: **end for**
12: **end for**
13: **end for**
14: **while** $D \neq \phi$ **do**
15: Let $d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)$ be the minimum in D
16: Generate a temporal cluster set: $U \leftarrow U_{(s_{N_1^{g_t}}, s_{N_2^{g_t}})} \cup U_{(s'_{N_1^{g_t}}, s'_{N_2^{g_t}})}$
17: Check the satisfaction of exclusive law for all different members $(\tilde{s}_{N_1^{g_t}}, \tilde{s}_{N_2^{g_t}}) \neq (\tilde{s}'_{N_1^{g_t}}, \tilde{s}'_{N_2^{g_t}}) \in U$
18: **if** the exclusive law was satisfied **then**
19: Update the clusters: $U_{(s_{N_1^{g_t}}, s_{N_2^{g_t}})} \leftarrow U$, $U_{(s'_{N_1^{g_t}}, s'_{N_2^{g_t}})} \leftarrow U$
20: **else if** $d_{min} = \infty$ **then**
21: $d_{min} \leftarrow d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)$
22: **if** $d_{min} < d_{th}$ **then**
23: **return** FALSE
24: **end if**
25: **end if**
26: **for** all $n \in \{R\} \cup \{N_1^{g_r}, N_2^{g_r} : g_r \neq g_t, \forall g_r \in \mathcal{G}\}$ **do**
27: $D \leftarrow D \setminus \{d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right)\}$
28: **end for**
29: **end while**
30: **for** all $(s_{N_1^{g_t}}, s_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}$ **do**
31: **if** rule for symbol-pairs in $U_{(s_{N_1^{g_t}}, s_{N_2^{g_t}})}$ does not exist in C_{g_t} **then**
32: $\left|\mathcal{M}_{C_{g_t}}\right| \leftarrow \left|\mathcal{M}_{C_{g_t}}\right| + 1$
33: Add a rule for symbol-pairs in $U_{(s_{N_1^{g_t}}, s_{N_2^{g_t}})}$ to C_{g_t}
34: **end if**
35: **end for**
36: **return** TRUE

and $(s'_{N_1^{g_t}}, s'_{N_2^{g_t}})$, the algorithm only considers the minimum distance among all nodes. Afterwards, it removes all distances that correspond to the same combination from D, as shown in lines 26–28 in Algorithm 1. After all the symbols are allocated (i.e. D becomes empty), we remove the repeated clusters to achieve the mapping $C_{g_t} : \mathcal{M} \times \mathcal{M} \rightarrow \mathcal{M}_{C_{g_t}}$. The complexity of this algorithm is $O\left(|\mathcal{M}|^4 |\mathcal{G}|\right)$.

IV. Node-Pair Grouping

The goal of node-pair grouping is to find the optimal partition $\mathcal{P} = \{\mathcal{G}_1, \ldots, \mathcal{G}_{|\mathcal{P}|}\}$. An optimal partition has the following characteristics: 1) node-pair(s) in each group $\mathcal{G} \in \mathcal{P}$ can perform joint encoding under a certain BER constraint, i.e. all $\mathcal{G} \in \mathcal{P}$ are feasible for joint encoding; 2) $|\mathcal{P}|$ must be the smallest among all feasible partitions (a feasible partition is a partition in which all groups are feasible). We also define a set $\mathcal{L} = \{\mathcal{P}_1, \ldots, \mathcal{P}_{|\mathcal{L}|}\}$, where $|\mathcal{L}| \geq 1$, which stores different partitions. From $\mathcal{L}$, an optimal partition can be selected.

A straightforward way of finding the optimal partition is exhaustive search, which enumerates all the possible partitions, neglects those partitions that contain infeasible groups, and chooses one partition with the minimum number of groups. However, when the total number of node-pairs increases, the number of possible partitions becomes very large. Hence, we propose a simplified method for node-pair grouping.

We note the following property.

Proposition 1: If group $\mathcal{G}$ is feasible, then any group $\mathcal{G}' \subseteq \mathcal{G}$ is also feasible.

Proof: Consider $g_t \in \mathcal{G}'$ and $\mathcal{G}' \subseteq \mathcal{G}$, according to the discussion in Section III-D, when constructing the encoding sub-function $C_{g_t}(\cdot)$ for group $\mathcal{G}$, the distance set is $D = \{d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right) : \forall (s_{N_1^{g_t}}, s_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}, \forall (s'_{N_1^{g_t}}, s'_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}, \forall n \in \{R\} \cup \{N_1^{g_r}, N_2^{g_r} : g_r \neq g_t, \forall g_r \in \mathcal{G}\}\}$; and when constructing $C_{g_t}(\cdot)$ for $\mathcal{G}'$, the distance set is $D' = \{d_n\left((s_{N_1^{g_t}}, s_{N_2^{g_t}}), (s'_{N_1^{g_t}}, s'_{N_2^{g_t}})\right) : \forall (s_{N_1^{g_t}}, s_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}, \forall (s'_{N_1^{g_t}}, s'_{N_2^{g_t}}) \in \mathcal{M} \times \mathcal{M}, \forall n \in \{R\} \cup \{N_1^{g_r}, N_2^{g_r} : g_r \neq g_t, \forall g_r \in \mathcal{G}'\}\}$. Because $\mathcal{G}' \subseteq \mathcal{G}$, it is easy to see that $D' \subseteq D$. If group $\mathcal{G}$ is feasible, then the minimum distance $d_{min} \in D$ between different clusters must be greater than or equal to the threshold d_{th}. Because the clustering procedure in Algorithm 1 starts with the minimum member in D (or, correspondingly, D') and attempts to arrange symbol-pairs into the same cluster when the exclusive law can be satisfied, if we remove some elements from D yielding D' (i.e. $D' \subseteq D$), we must have $d'_{min} \geq d_{min} \geq d_{th}$, where d'_{min} is the minimum distance among different clusters when considering the distance set D' and the group $\mathcal{G}'$. Hence $\mathcal{G}'$ is also feasible. ■

Based on Proposition 1, we start the partitioning with finding a set of small-size feasible groups and then extend some of them by group splitting and re-combining in order to reduce the total number of the feasible groups in the partition. The process of partitioning includes two steps. In the first step, we attempt to find a basic partition, in which each group only contains one or two node-pair(s). In the second step, we attempt to reduce the number of the groups in the partition obtained in the first step, via group splitting and re-combining.

The partitioning in the first step can be formulated by a graph model, where vertices in the graph represent the node-pairs. If two node-pairs can constitute a feasible group, we connect the two corresponding vertices with an edge. This checking process requires running Algorithm 1 with $|\mathcal{G}| = 2$ among all possible node-pairs, hence its complexity is $O\left(|\mathcal{M}|^4 K^2\right)$. Then, finding the optimal partition, in which the maximum number of node-pairs in each group is two, is equivalent to finding the maximum matching of the graph, which can be solved by the blossom algorithm with complexity $O\left(K^{2.5}\right)$ [10]. Based on the matching result, the two node-pairs that are connected by an edge in the matching are allocated into the same group, and we get our desired partition which includes multiple groups of one or two node-pair(s).

In the second step, we try to further reduce the number of groups by multiple group splitting and re-combining rounds, as shown in Algorithm 2. We save the optimal partition in each round (including the result from the first step) in $\mathcal{P}_\mathcal{T}$. In each round, we first select the group(s) with the minimum number of node-pair(s), and save such group(s) in $\mathcal{P}'_\mathcal{T}$. We split each group $\mathcal{G}$ in $\mathcal{P}'_\mathcal{T}$ and merge each node-pair in $\mathcal{G}$ into the remaining groups $\mathcal{P}_\mathcal{T} \setminus \{\mathcal{G}\}$ to form new feasible partitions. All the new partitions have the same number of groups, which is one less than that in the last round, i.e. $|\mathcal{P}_\mathcal{T}|-1$, and we save all the feasible new partitions in $\mathcal{L}$. Because a group containing larger number of node-pairs also has larger probability of decoding error (due to error propagation when using network coding), we select the partition in which the number of node-pairs are relatively evenly distributed across each group. This can be achieved by calculating the variances σ^2 of the number of node-pairs in each group of each partition in $\mathcal{L}$ and choosing the partition with the minimum σ^2. Then, $\mathcal{P}_\mathcal{T}$ is updated to store the new partition with minimum σ^2. The same operation of group splitting and re-combining will be executed on the selected partition $\mathcal{P}_\mathcal{T}$ in the next round. The second step is finished when no more new partition is produced or only one group is included in the selected partition $\mathcal{P}_\mathcal{T}$. If we assume that the minimum group found in Line 6 has at most K' node-pairs, the complexity of the algorithm is $O\left(K^3 |\mathcal{M}|^4 \left(\frac{K}{K'}-1\right)^{K'}\right)$. If we allow only a limited number of node-pairs in each group, then K' is a constant.

V. Simulation Results

We evaluate the performance of our proposed scheme in a network with one relay and K node-pairs. All the end nodes are randomly distributed in a 200×200 m^2 square region and the relay is placed in the center. We use Rician flat-fading channel with Rician factor 5 dB. The noise power density is –174 dBm/Hz and the receiver bandwidth is 1 MHz. The channel power gain is calculated by $1/d^4_{n_a n_b}$, where $d_{n_a n_b}$ is the physical distance between nodes n_a and n_b in meters. We consider QPSK and 16QAM modulations with different number of node-pairs or different transmission power. Each setting is run with 50 different random seeds and the overall performance is plotted. The maximum BER threshold is set to 10^{-3}, and the corresponding d_{th} is evaluated according to digital communication theory [9], by considering the worst-case BER between closest neighboring constellation points. We compare the performance with the exhaustive search method for node grouping.

Figs. 3 and 4 show the average throughputs for node-pairs which can perform PNC (i.e., in throughput calculation, we neglect those node-pairs which happen to be in bad channel status and cannot perform PNC within the BER requirement). The throughput is expressed as the number of packets transmitted in each timeslot. We also show the percentage of node-pairs for which PNC can be performed (denoted as PNC ratio), which is the same for different methods because it only depends on the channel condition. In

Algorithm 2 Node-Pair Grouping

1: Define three temporal sets: $\mathcal{P}_\mathcal{T}$, $\mathcal{P}'_\mathcal{T}$ and $\mathcal{L}_\mathcal{T}$
2: Generate a partition in which each group only contains one or two node-pair(s) using maximum matching, and save it in $\mathcal{P}_\mathcal{T}$
3: **do**
4: Delete all the partitions in $\mathcal{L}$: $\mathcal{L} \leftarrow \phi$
5: **if** $|\mathcal{P}_\mathcal{T}| > 1$ **then**
6: Find $\mathcal{P}'_\mathcal{T} \subseteq \mathcal{P}_\mathcal{T}$ where $\mathcal{P}'_\mathcal{T}$ contains the group(s) of least number of node-pairs
7: **for all** $\mathcal{G} \in \mathcal{P}'_\mathcal{T}$ **do**
8: Put node-pairs in $\mathcal{G}$ into groups in $\mathcal{P}_\mathcal{T} \setminus \{\mathcal{G}\}$, achieve $(|\mathcal{P}_\mathcal{T}|-1)^{|\mathcal{G}|}$ new partitions saved in $\mathcal{L}_\mathcal{T}$
9: **for all** $\mathcal{P} \in \mathcal{L}_\mathcal{T}$ **do**
10: **if** $\mathcal{P}$ is not feasible for joint encoding **then**
11: $\mathcal{L}_\mathcal{T} \leftarrow \mathcal{L}_\mathcal{T} \setminus \{\mathcal{P}\}$
12: **end if**
13: **end for**
14: $\mathcal{L} \leftarrow \mathcal{L} \cup \mathcal{L}_\mathcal{T}$
15: **end for**
16: **if** $\mathcal{L} \neq \phi$ **then**
17: **for all** $\mathcal{P} \in \mathcal{L}$ **do**
18: Compute the variance $\sigma^2_\mathcal{P} = \frac{1}{|\mathcal{P}|}\sum_{i=1}^{|\mathcal{P}|}(|\mathcal{G}_i|-\mu)^2$ where $\mu = \frac{1}{|\mathcal{P}|}(|\mathcal{G}_1|+\cdots+|\mathcal{G}_{|\mathcal{P}|}|)$
19: Put it in the set: $\mathcal{V} \leftarrow \mathcal{V} \cup \sigma^2_\mathcal{P}$
20: **end for**
21: Let $\sigma^2_\mathcal{P}$ be the minimum member in $\mathcal{V}$, put the corresponding partition in temporal set $\mathcal{P}_\mathcal{T} \leftarrow \mathcal{P}$
22: **end if**
23: **end if**
24: **while** $\mathcal{L} \neq \phi$
25: **return** $\mathcal{P}_\mathcal{T}$

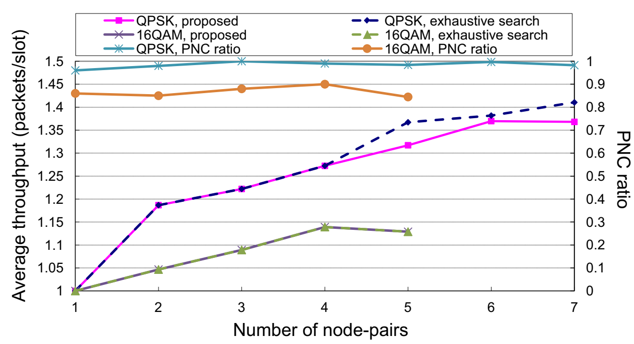

Fig. 3. Average throughput for node-pairs which can perform PNC and PNC ratio versus number of node-pairs.

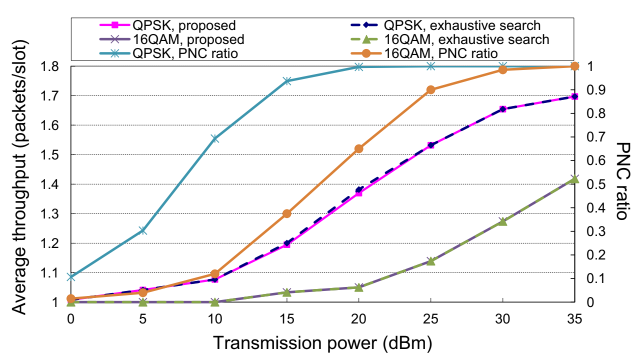

Fig. 4. Average throughput for node-pairs which can perform PNC and PNC ratio versus transmission power.

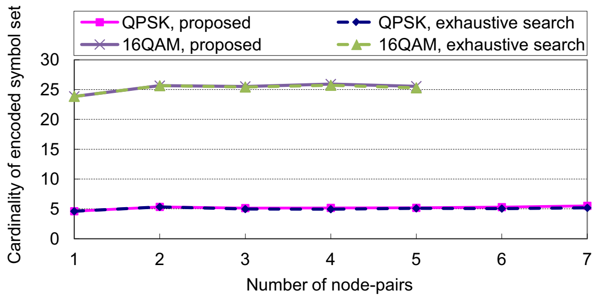

Fig. 5. Average cardinality of encoded symbol set versus number of node-pairs.

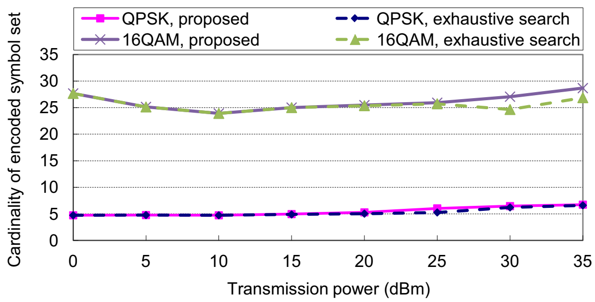

Fig. 6. Average cardinality of encoded symbol set versus transmission power.

Fig. 3, we consider scenarios with different number of node-pairs, and the transmission powers are respectively set to 20 dBm and 25 dBm for QPSK and 16QAM. We can observe that the performance of our proposed scheme is close to the exhaustive search method which returns the optimal value. When the number of node-pairs is large, the proposed scheme slightly underperforms the exhaustive search scheme, because we use a heuristic (suboptimal) node-pair grouping method when the number of node-pairs in the group exceeds two. The minimum throughput is 1 packet/slot, and is achieved when all the feasible groups contain only one node-pair, which is the same as conventional PNC without joint encoding. The throughput increases with the number of node-pairs because the joint encoding opportunity increases. When using 16QAM, the throughput stops increasing when the number of node-pairs is larger than four, because it has reached the upper limit of the number of jointly encoded node-pairs at 25 dBm transmission power, and therefore we did not simulate cases with more than five node-pairs. In Fig. 4, the throughput is evaluated when transmission power varies from 0 dBm to 35 dBm. The number of node-pairs is set to 6 for QPSK modulation, and 4 for 16QAM. When the transmission power is high, the average throughput with QPSK approximately reaches the upper bound of 1.714 packets/slot, which is achieved when all the six node-pairs compose one single feasible group. It can be observed that in all cases, PNC with joint encoding outperforms PNC without joint encoding (which corresponds to the bottom-line throughput of 1 packet/slot).

When executing joint encoding, the encoded symbol set $\mathcal{M}_C$ may be larger than the original symbol set $\mathcal{M}$. Figs. 5 and 6 show the average cardinality of the encoded symbol set. We can observe that the average cardinality ranges from 4 to 7 (with the majority close to 5) for QPSK and from 20 to 30 (with the majority close to 25) for 16QAM. Note that such an increase in the cardinality is also common for PNC without joint encoding, and sphere packing can be used to generate constellations for irregular symbol sets, as discussed in [7].

VI. Conclusions

In this paper, we have proposed a joint encoding and node-pair grouping scheme. Joint encoding is achieved with a two-level encoding function. The low-level sub-function processes the simultaneously transmitted symbol-pair and maps them into an encoded symbol, and the high-level sub-function further encodes these symbols digitally. The node-pair grouping process attempts to find the feasible partition with the least number of groups, thereby minimizing the number of necessary timeslots. It first finds the partitions with a maximum of two node-pairs in each group, using maximum matching in a graph. Then, it attempts to split and re-combine groups, starting from the smallest group, to further reduce the total number of groups. Simulation results show that the proposed scheme can improve the throughput compared with conventional PNC without joint encoding, and the results when using the proposed scheme are close to the optimum. Future work can focus on reducing the complexity of algorithms as well as reducing the cardinality of the encoded symbol set.

Acknowledgment

This work was supported in part by the National Natural Science Foundation of China (61172051), the Fundamental Research Funds for the Central Universities (N110204001, N110804003, N120804002, N120404001), the Program for New Century Excellent Talents in University (NCET-12-0102), the Specialized Research Fund for the Doctoral Program of Higher Education (20110042110023, 20110042120035, 20120042120049), and Liaoning Province PhD Start-up Fund (20131029).

References

[1] S. Zhang, S. C. Liew, and P. P. Lam, "Hot topic: Physical-layer network coding," in *Proc. ACM MobiCom 2006*, Sep. 2006, pp. 358–365.

[2] Y. Huang, S. Wang, Q. Song, L. Guo, and A. Jamalipour, "Synchronous physical-layer network coding: a feasibility study," *IEEE Trans. Wireless Commun.*, Apr. 2013, accepted for publication.

[3] S. Wang, Q. Song, X. Wang, and A. Jamalipour, "Rate and power adaptation for analog network coding," *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2302–2313, Jun. 2011.

[4] M. Hekrdla and J. Sykora, "Optimised constellation prerotation for 3-terminal 1-relay network with wireless network coding," *IEEE Commun. Lett.*, vol. 16, no. 8, pp. 1200–1203, Aug. 2012.

[5] J. H. Sorensen, R. Krigslund, P. Popovski, T. K. Akino, and T. Larsen, "Scalable denoise-and-forward in bidirectional relay networks," *Computer Networks*, vol. 54, no. 10, pp. 1607–1614, Jul. 2010.

[6] S. Wang, Q. Song, L. Guo, and A. Jamalipour, "Constellation mapping for physical-layer network coding with M-QAM modulation," in *Proc. IEEE GLOBECOM 2012*, Dec. 2012, pp. 4429–4434.

[7] T. Koike-Akino, P. Popovski, and V. Tarokh, "Optimized constellations for two-way wireless relaying with physical network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, Jun. 2009.

[8] S. Wang, Q. Song, X. Wang, and A. Jamalipour, "Distributed MAC protocol supporting physical-layer network coding," *IEEE Trans. Mobile Comput.*, vol. 12, no. 5, pp. 1023–1036, May 2013.

[9] R. Gallager, *Principles of Digital Communications.* Cambridge: Cambridge University Press, 2008.

[10] S. Micali and V. V. Vazirani, "An algorithm for finding maximum matching in general graphs," in *Proc. of 21st Annual Symposium on Foundations of Computer Science*, Oct. 1980, pp. 17–27.