Adaptive Relaying Method Selection for Multi-Rate Wireless Networks with Network Coding

Fanzhao Wang, Shiqiang Wang, *Student Member, IEEE*, Qingyang Song, *Member, IEEE*, and Lei Guo, *Member, IEEE*

***Abstract*—To maximize the throughput of multi-rate wireless networks, in this letter, we propose a scheme to adaptively select the relaying method among analog network coding (ANC), conventional network coding (CNC), no relaying (i.e. direct transmission without relaying), and plain routing. We first discuss the achievable data rates with different relaying methods under specific channel conditions, and then propose an algorithm with polynomial-time complexity that provides a sub-optimal solution to the relaying method selection problem. Simulation results show that the proposed scheme can effectively improve the network throughput compared with existing schemes, and its performance is near to the optimal performance. The results in this letter also provide some insights for the design of routing protocols in the future.**

***Index Terms*—Cooperative communications, multi-rate, network coding, relaying method, wireless networks.**

I. INTRODUCTION

NETWORK coding has promising performance in wireless relay networks [1]. It allows the packets to be encoded at the relay before they are forwarded to the destinations, which reduces the required transmission time compared with plain routing schemes, and increases the throughput. As an extension to conventional network coding (CNC), analog network coding (ANC) further increases the bandwidth efficiency by making use of simultaneous transmissions [2].

However, it is not always possible to perform network coding. The opportunities of performing network coding depend on the network topology, channel conditions, etc. Existing schemes choose network coding as the relaying method whenever there is coding opportunity, and ANC has higher priority compared with CNC (i.e. when ANC and CNC can both be performed, ANC is selected) [3], [4]. These methods are suitable for single-rate networks, while in multi-rate networks, the situation becomes more complicated and simple judgments based on coding opportunities are inefficient. Some existing works attempt to dynamically select the best relaying method among CNC and plain routing [5]. Others try to optimize the data rate of ANC [6] or CNC [7]. However, to the best of our knowledge, the concurrent support of ANC, CNC, no relaying (i.e. direct transmission without relaying), and plain routing has not been adequately studied in the literature.

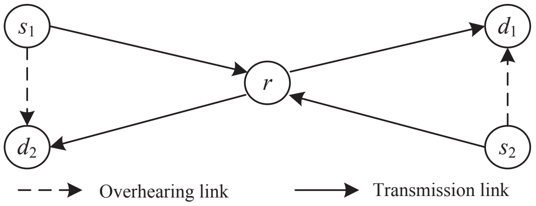

Fig. 1. An example network with one relay node and two sessions.

In this letter, we focus on the selection of different relaying methods in multi-rate networks. The relaying methods include ANC, CNC, no relaying, and plain routing. We consider wireless relay networks with one relay and multiple source/destination pairs, where node r is the relay, s_i and d_i ($i = 1, 2, 3, \ldots$) are respectively the source nodes and (their corresponding) destination nodes, as shown in Fig. 1. We refer to the node pair $s_i \to d_i$ as a *session*. The relay coordinates the transmission of different sessions, and selects the appropriate relaying methods to maximize the data rate. This network topology can be regarded as a network with one access point (or base station) and multiple surrounding nodes that wish to transmit information with each other. The access point (or base station) coordinates the transmission of the source nodes. It can also be regarded as a part of a general multi-hop network, in which potential relays temporarily perform as coordinators and decide which relaying method to use, such as in the medium access control (MAC) protocol proposed in [3].

II. ACHIEVABLE DATA RATES

This section summarizes the achievable data rates when using the aforementioned four different relaying methods in multi-rate networks with asymmetric links. Considering fairness issues, we assume that only one packet in each session is transmitted within one scheduling round[1] and the packet lengths of all sessions are identical. By this means, each session attains the same average data rate, which is the same fairness measure as used in [7]. Note that the time that each session consumes to transmit a packet can be different. Let T_Σ denote the total time required for transmitting one packet from each of the K sessions. Then, the achievable data rate can be evaluated by:

$$R = K/T_\Sigma\,. \tag{1}$$

It follows that given the total number of sessions K, the average data rate R only depends on the necessary transmission time of each session.

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F. Wang, Q. Song (corresponding author), and L. Guo are with the School of Information Science and Engineering, Northeastern University, Shenyang 110819, P. R. China (e-mails: fanzhaowang@gmail.com, {songqingyang, guolei}@ise.neu.edu.cn).

S. Wang is with the Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ, United Kingdom (e-mail: shiqiang.wang11@imperial.ac.uk).

[1] The scheduling round means the time when all the sessions have been served once.

In the following, we analyze the average transmission time for each session, when using different relaying techniques. For ANC and CNC, we group several sessions together because packets in these sessions are encoded at the relay and then simultaneously broadcasted to the destinations.

A. ANC

As shown in [8], only two sessions can generally be encoded with ANC, unless using sophisticated self-interference cancelation techniques. According to [8], the average transmission time for each session performing ANC is:

$$T_{ANC} = 1/\min\left\{C_{s_1,d_2}, C_{s_2,d_1}, C_{s_1,r,d_1}, C_{s_2,r,d_2}\right\}\ , \tag{2}$$

where the sessions $s_1 \rightarrow d_1$ and $s_2 \rightarrow d_2$ are the two sessions that perform ANC, $C_{s_i,d_j} (i, j = 1, 2, 3, ...)$ denotes the capacity of the link from node s_i to node d_j, and C_{s_i,r,d_i} denotes the capacity of the amplify-and-forward link from s_i to d_i (amplified and forwarded by the relay r). The minimum operator in (2) is due to the fact that the source nodes transmit simultaneously and the relay does not decode the packets. The source nodes have to transmit at the minimum data rate that can be carried on all the links, to ensure decodability [8].

B. CNC

Unlike ANC, CNC can encode multiple packets into one coded packet at the relay. Suppose sessions with index numbers 1 to k are encoded, the average transmission time of CNC can be written by:

$$T_{CNC}=\frac{1}{k}\left(\frac{1}{\min\{\cup_j C_{r,d_j}\}}+\sum_{i=1}^{k}\frac{1}{\min\left\{C_{s_i,r},\cup_{j\neq i}C_{s_i,d_j}\right\}}\right). \tag{3}$$

The first minimum operator in (3) is because the relay broadcasts the encoded packet to all the destinations. Note that the full rate broadcasting scheme proposed in [9] relaxes to minimum rate in our scenario, because we consider packets with the same length and having some sessions transmitting at a higher data rate will only cause those sessions stop transmitting earlier, which does not increase the actual average data rate. The second minimum operator is because, for every packet, the relay and all the destinations other than the intended destination have to overhear the packet, so that the intended destination can successfully decode the packet.

C. Plain Routing

In plain routing, since all transmissions are independent and coding is not performed, the transmission of one packet requires two time slots and the data rates of the involved links can reach their corresponding capacities. Therefore, the transmission time for one session is:

$$T_{PR} = 1/C_{s_1,r} + 1/C_{r,d_1}\ . \tag{4}$$

D. No Relaying

In the case of direct transmission without relaying, for one session, the transmission time is:

$$T_{NR} = 1/C_{s_1,d_1}\ . \tag{5}$$

III. ADAPTIVE RELAYING METHOD SELECTION

From the above analysis, it is obvious that network coding may not always perform well because the data rates has to be adjusted based on the worst-quality link. In this section, we propose an adaptive relaying method selection scheme to maximize the network throughput.

A. Problem Formulation

As aforementioned, the relay coordinates the transmission of different sessions, and in each scheduling round, one packet from each session is sent. To enable coordination, the relay node maintains a queue $\mathbf{P}$ with size n which stores the essential information of the first packet in each session. This queue is similar with the virtual queue in [3]. The size n corresponds to the number of sessions that currently have packets to send.

Our problem is to match each packet in $\mathbf{P}$ with an appropriate relaying method, to maximize the throughput. We also need to group the packets to see whether ANC or CNC should be selected.

B. Optimal Solution via Exhaustive Search

We note that when using different relaying methods, different number of packets can be involved. Particularly when using CNC, the number of encoded packets may range from two to the total number of packets (n). Therefore, in order to find the appropriate relaying method for each session, we need to enumerate all the possible partitions of the packet set $\mathbf{P}$. For every partition, we assign each subset a relaying method that requires the minimum transmission time (and hence has the highest data rate) to transmit the packets in the subset[2]. Then, the specific partition and the corresponding relaying methods that provides maximum throughput will be selected for actual transmission. The total number of partitions of a set is the Bell number B_n, which can be approximated by $\left(\frac{n}{\mathrm{e}\ln n}\right)^n$ for sufficiently large n [10].

C. Sub-Optimal Solution via A Simplified Method

In order to make the optimization feasible, we propose a simplified method that finds a suboptimal solution to the throughput maximization problem. The main idea is that, instead of enumerating all the partitions, we only consider partitions in which all the subsets have a size of two, i.e. *pairwise partitions*[3]. By this means, we can cover the cases of ANC, no relaying, plain routing, and also most cases of CNC. To incorporate with CNC with multiple (three or more) encoded packets, we heuristically attempt to encode more packets with CNC afterwards.

1) Pairwise Partition and Optimization: The pairwise partition and best relaying method selection process can be formulated by a graph model. We construct a complete weighted graph $G(V, E)$ in which vertices V correspond to packets in $\mathbf{P}$. We set the weight ω_e of an edge $e \in E$ as the minimum

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[2] Also note that the number of packets in the subset must also match with the requirement of the relaying method. For instance, a subset that contains three or more packets can only be assigned to CNC.

[3] We assume that there are always even number of packets in $\mathbf{P}$. When the number of packets is odd, the assumption is easily satisfied by adding a packet without transmission time.

Algorithm 1 Adaptive Relaying Method Selection Scheme

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1: Initiate a complete weighted graph $G(V, E)$ with vertices $V$
   corresponding to packets in $\mathbf{P}$.
2: Calculate the weight $\omega_e$ for each $e \in E$ according to (6), and store
   the relaying method(s) corresponding to the min. transmission time.
3: Perform minimum-weight perfect matching (MWPM) for $G$.
4: Attempt to encode more packets for packet pairs with CNC.
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transmission time of the two sessions corresponding to the two vertices of e, i.e.

$$\begin{aligned}\omega_e = \min\{&2T_{ANC}, 2T_{CNC}, T_{PR1} + T_{PR2},\\ &T_{PR1} + T_{NR2}, T_{NR1} + T_{PR2}, T_{NR1} + T_{NR2}\}. \quad (6)\end{aligned}$$

The factor 2 comes from the fact that two sessions are involved, and also note that there are four possible combinations for no relaying and plain routing. The relaying method(s) corresponding to the minimum transmission time is the optimal relaying method for the two vertices (and hence the corresponding packets) that are connected by the edge. However, because each vertex is generally connected to multiple edges, we still need to find the best set of edges that will be used in relaying.

The remaining problem is equivalent to the problem of minimum-weight perfect matching (MWPM) for G, which can be formulated as the following binary integer program:

$$\begin{aligned}\min_{x_e} \quad & \sum x_e \omega_e \\ s.t. \quad & x_e \in \{0, 1\}, \forall e \in E, \\ & x\left(\delta\left(v\right)\right) = 1, \forall v \in V, \end{aligned} \qquad (7)$$

where, $\delta(v)$ denotes the set of edges that meet the node v, and $x\left(\delta(v)\right) = \sum_{e \in \delta(v)} x_e$. A perfect matching of graph G is a subset of edges such that each node in G is met by exactly one edge in the subset. MWPM is to find a perfect matching $M \subseteq E$ with minimum sum-weight, which can be solved by Edmonds' blossom algorithm [11]. The matching result corresponds to the optimal pairwise partition; and every packet pair corresponding to the edges in M, together with the relaying method(s) that achieves the minimum in (6), is an optimal solution to the relaying method selection problem when the maximum number of encoded packets is two.

2) Encoding Multiple Packets with CNC: Because it is possible to encode three or more packets with CNC, when the optimal relaying method for a pair of packets is CNC, we should attempt to encode more packets together. We solve this problem with a heuristic method. We start with the first packet pair for which CNC is selected, and enumerate the remaining packets and test whether encoding other packets together will reduce the total transmission time. If yes, these packets will be encoded together, and the original relaying method allocation will be updated. Afterwards, the remaining packets pairs with CNC are processed in the same manner.

3) Complexity: The main steps of the proposed scheme are shown in Algorithm 1. The proposed scheme first calculates the weights of all edges, which needs $H = \binom{2}{n} = \frac{n(n-1)}{2}$ calculations. Then, MWPM is performed, with complexity $O(n(H + n \log n))$ [11]. Finally, for packet pairs with CNC, we need to attempt to encode more packets. Because there are at most $n/2$ packet pairs, and each packet pair tests at most $n - 2$ other packets to see whether it is beneficial to encode them together, the complexity of this process is $O(n^2)$. Hence, the total complexity of the proposed scheme is $O(n(H + n \log n)) + O(n^2) = O(n^3)$, i.e. it is polynomial.

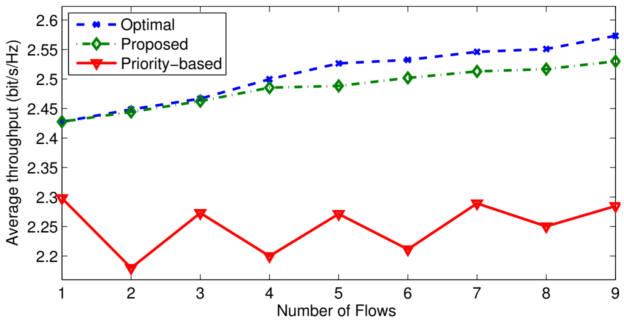

Fig. 2. Throughput vs. the number of flows including no relaying.

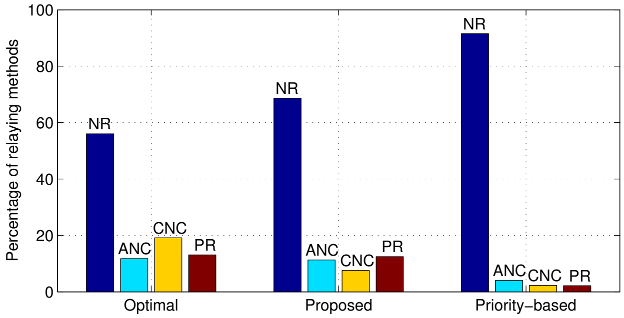

Fig. 3. Percentages of selected relaying methods including no relaying, with eight flows. "NR" stands for "no relaying", "PR" stands for "plain routing".

IV. SIMULATION RESULTS

We evaluate the performance of the proposed relaying method adaption scheme in a network with one relay node and 15 end nodes via simulations. The end nodes are uniformly distributed in a 500×500 m^2 square region; and the relay is placed in the center of the region. Some end nodes are randomly selected to generate unicast flows to random destinations. The maximum transmission power of each node is 5 dBm. We consider a Rician flat-fading channel with Rician factor $K = 5$ dB. The noise power density is set to –174 dBm/Hz, the receiver bandwidth is 1 MHz, and the noise figure is 6 dB. Each simulation was run with 10,000 different random seeds (which correspond to 10,000 different network topologies and session set-ups) to obtain the overall performance. However, note that the set of network topologies and session set-ups are identical over different relaying method adaptation schemes under evaluation.

We compare the proposed sub-optimal scheme with the optimal exhaustive search scheme, and also with a scheme that selects the relaying methods in a priority order, with no relaying as the highest priority, following with ANC (second), CNC (third), and plain routing (lowest). This priority-based scheme resembles existing schemes that selects ANC or CNC whenever there is network coding opportunity. The no relaying method has the highest priority because it corresponds to the case where relaying has not been selected by the routing protocol. We regard 1 bit/s/Hz as the basic data rate, any data rate above this value is regarded as feasible for transmission.

Fig. 2 shows the throughput with different relaying selection schemes, under different number of flows. It can be observed that the proposed scheme offers nearly optimal performance,

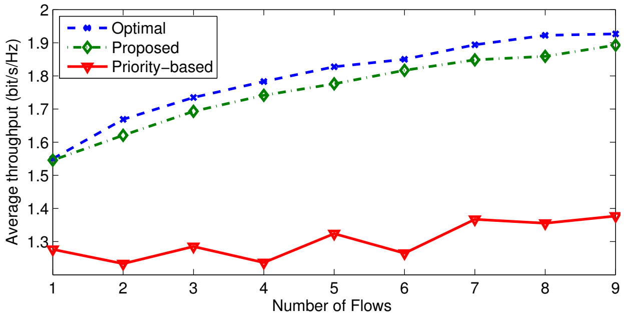

Fig. 4. Throughput vs. the number of flows excluding no relaying.

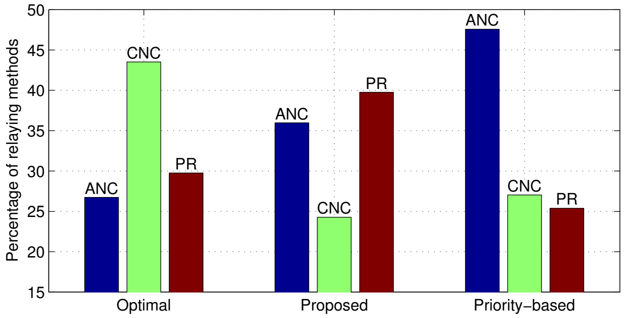

Fig. 5. Percentages of selected relaying methods excluding no relaying, with eight flows. "PR" stands for "plain routing".

with throughput gain ranging from 5% to 13% compared with the priority-based scheme. Fig. 3 shows the percentages of different relaying methods that have been selected, when the number of flows is eight. It can be observed that, due to the random placement of nodes, a large number of sessions manage to transmit without relaying. When we only look at the percentages of no relaying, it is also interesting to find out that the percentage is the highest with the priority-based scheme, and lowest with the optimal scheme. Meanwhile, the percentages of the other relaying methods tend to be higher in the optimal and proposed schemes, compared with the priority-based scheme. This is because the priority-based scheme selects the relaying method with the highest priority (i.e. no relaying) whenever the link quality allows basic data transmission, resulting in a higher percentage of no relaying.

To further investigate the cases when relaying is necessary, we remove the data when no relaying is selected from our statistics. By this means, we get the performance for scenarios where relaying is needed, or where better performance can be obtained with relaying. The results are shown in Figs. 4 and 5. It can be observed that the throughput of the proposed scheme is still similar with the optimal scheme, and the throughput gain of the proposed scheme over the priority-based scheme ranges from 21% to 43%. Similar trends exist with the percentages as in the case *with* no relaying. The only difference is that the highest priority scheme is ANC now. It can also be observed that the optimal scheme leads to a substantially higher ratio of CNC compared with the proposed scheme. This is because the proposed scheme is based on pairwise matching and only utilizes CNC with multiple packets in a heuristic approach.

It can also be observed in Figs. 2 and 4 that the throughput of the priority-based scheme fluctuates with the number of flows. In particular, an even number of flows leads to lower throughput. This is mainly because the priority-based scheme performs network coding whenever there is a coding opportunity, without judging whether network coding increases the data rate compared with plain routing. When the number of flows is even, there tends to be more coding opportunity; and because coding has not been optimized, the throughput is reduced.

From Figs. 3 and 5, we can also observe that the percentage of CNC is higher in the optimal scheme compared with other schemes. This is mainly because the case of encoding multiple packets with CNC can be better exploited with the optimal scheme, while the proposed scheme only attempts to encode multiple packets heuristically.

V. CONCLUSION

In this letter, we have been focusing on integrating different relaying methods in multi-rate wireless networks. We have proposed an adaptive relaying method selection scheme with polynomial-time complexity, to find the appropriate relaying method for sessions sharing a common coordinator which may or may not act as a relay. Simulation results show that the proposed scheme performs close to the optimal exhaustive search scheme, and outperforms the existing priority-based scheme.

The results in this letter also imply that conventional routing schemes may not work well in multi-rate networks with network coding. In particular, optimal routing cannot be found by simply adding the costs of individual links. The joint effects of links need to be considered, and more sophisticated cost functions need to be designed to support the different relaying methods. The extension of our work to general multi-hop scenarios and routing protocols will be considered in the future.

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